Sling psychrometry: relations among T_{wet}, T_{air}, T_{dew}, P_{air}, and humidity

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The method of sling psychrometry is to make the boundary layer conductance at the two thermometer bulbs to be so large that only two terms matter in the energy balance: evaporative cooling, Q_E, and convective heat transfer, Q_C. At steady state, the sum of these two terms is zero, which we may write as

$$0 = -\frac{\lambda g_b}{P}[e_s(T_{wet}) - e_a] - g_b C_P (T_{wet} - T_{air}] \qquad (1)$$

Here,

- λ = molar heat of vaporization of water (about 44,000 J mol^{-1})
- g_b = boundary-layer conductance, in molar units, as commonly used in plant physiology (mol m^{-2}s^{-1})
- P = total air pressure
- $e_s(T_{wet})$ = saturated vapor pressure of water (Pa) at the wet-bulb temperature
- T_{wet} = wet-bulb temperature
- e_a = partial pressure of water vapor in ambient air
- C_P = molar heat capacity of air (about 29 J mol^{-1})

We can factor out g_b and rearrange Eq. (1) to

$$T_{air} - T_{wet} = \frac{\lambda}{PC_P}[e_s(T_{wet}) - T_{air} \qquad (2)$$

For a first approximation, we can express $e_s(T_{wet})$ using a linear variation of saturating vapor pressure with temperature:

$$e_s(T_{wet}) = e_s(T_{air}) + e'[T_{wet} - T_{air}] \qquad (3)$$

We can use a quadratic form for higher accuracy, but the principle remains the same.

Substituting Eq. (3) into Eq. (2), we obtain

$$T_{air} - T_{wet} = \frac{\lambda}{PC_P}[e_s(T_{air}) - e_a - e' \cdot (T_{air} - T_{wet})] \qquad (4)$$

We can bring all the terms in temperature to the left-hand side, to obtain

$$[T_{air} - T_{wet}][1 + \frac{\lambda e'}{PC_P}] = \frac{\lambda}{PC_P} e_s(T_{air})\,[1 - h_r] \qquad (5)$$

The factor on the right-hand side after $\lambda/(PC_P)$ is just the vapor-pressure deficit, D. I write relative humidity, h_r, as a fraction (40% = 0.4, literally). Clearly, we can solve for the temperature difference as

$$T_{air} - T_{wet} = \frac{\lambda D}{PC_P[1 + \frac{\lambda e'}{PC_P}]} \tag{6}$$

Here is a numerical example:

$T_{air} = 30°C \rightarrow e_s(T_{air}) = 4246$ Pa, e' = 250 Pa K^{-1}
$h_r = 0.5 \rightarrow e_a = 2123$ Pa
$P = 10^5$ Pa
λ, C_P as above.

Substituting these numerical values into Eq. (5) yields $T_{air} - T_{wet} = 6.72°C$.

We can also solve for the dew-point temperature, T_{dew}. Again using the linear approximation for the dependence of vapor pressure on temperature, we have

$$e_s(T_{dew}) = e_s(T_{air}) - e'[T_{air} - T_{dew}] \tag{7}$$

The order of temperatures was reversed in the last term to obtain a positive number. We can solve this as

$$T_{air} - T_{dew} = \frac{e_s(T_{air}) - e_a}{e'} \tag{8}$$

For the case just considered, this equates to 8.49°C. The wet-bulb T cannot descend to the dew point; the evaporative heat loss would be zero at $T = T_{dew}$ and could not balance the convective heat gain.

I'm sure there are derivations of this in texts, but none that I can find at hand. The principle is the same in any derivation.